# Is there mathematical concepts that are real?

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According to [3], C. F. Gauss said: If $e^{i\pi} = -1$ was not immediately apparent to a student upon being told it, that student would never become a first-class mathematician. We will explore the arguments that support Gauss's claim in order to prove that there are no mathematical concepts that are real in Steiner's sense.

We conform to the position that concept exists if it satisfies the W. O. Quine's condition: $Fs$ exist if $\exists x Fx$ is a theorem of a true theory; cf. [8]. But M. Steiner claims in [10] that it is possible for $Fs$ to satisfy this condition without being real. His inspiration is P. Bridgman's definition of physical reality: Something is physically real if it is connected with physical phenomena independent of those phenomena which entered its definition; cf. [1] p.56.

There is something profoundly right in the idea that the real is that which has properties transcending those which enter its definition and Steiner's aim is to show that mathematical entities can occasionally be said to be real in exactly the same sense.

Quine's condition is applicable to the existence of mathematical entities: scientific theories are committed to the existence of mathematical entities, and since we regard some of them as true, we must regard mathematical entities as existent. However, according to Steiner, this is not an argument for the reality of mathematical entities.

To demonstrate the reality of an entity in the natural sciences one typically shows that the entity is indispensable in explaining some new phenomenon. In this way the entity acquires new and independent descriptions. Steiner applies the same idea in mathematics.

For example, $\pi$ is real because we have at least two independent descriptions for $\pi$. Geometric, $\pi = \frac{C}{2r}$ and analytic, $\pi = \frac{\ln(-1)}{i}$. In the first case $\pi$ is derived from the formula for the circumference of a circle $C$ with radius $r$. In the second case $\pi$ is derived from the special case of Euler's formula, $e^{pii} = -1$.

We know by deductive proof that the descriptions are coreferential (unlike the situation in the physical sciences where this is demonstrated empirically). But then, how can provably coreferential descriptions be regarded as independent? Steiner's answer is to distinguish between two kinds of proof of coreference in mathematics: those which are nonexplanatory and merely demonstrate the coreference, and those which explain it. Descriptions are independent if the proofs of their coreferentiality are nonexplanatory.

We show that the “independence of the descriptions of two mathematical entities” is not additionally explained by the “absence of explanatory proofs of their coreference”, so we will stick with “independence” as a less vague criterion.

After a detailed analysis of the “reality status” of $\pi$, in the previously described context, we conclude that $\pi$ is not real in Steiner’s sense. As a matter of fact, it is difficult to prove for any mathematical concept that it is real in Steiner’s sense. Namely, it is not enough to formulate two descriptions of a concept and find a proof of their coreference which keeps the descriptions independent. It should be proved that all proofs of their coreference are such.

But mathematical theories are deeply connected and in the entire history of mathematics, mathematicians are constantly striving to discover these connections. For example, it is typical for mathematicians to persistently search for new proofs of old theorems in order to discover these intertheoretical dependencies.

Hence, our hypothesis is that no mathematical concept is real in Steiner’s sense.

# References


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